Challenges and Strategies in Transformer Design

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Outline of Presentation

• Transformer technology: overview and perspective
• Challenges and issues in design of
  • Magnetic circuit
  • Windings
  • Insulation
• Stray loss evaluation and control
• Short circuit withstand
• Recent trends in computations
Transformer Technology: Overview and Perspective

- Transformer: a mature product – 100 years old

- Technology hasn’t changed drastically, but the challenges are:
  - Continuous increase in ratings and sizes
  - Limitations on weight and space
  - Global market - competition
  - Accurate prediction of performance parameters
  - Increasing power system complexities

- Increase in voltage class, rating and size requires
  - Updating of design baseline
  - Upgradation of manufacturing technology
  - Strict process control
Requirement of modern power transformers

- Accurate electromagnetic field calculations
  - Minimization of eddy and stray losses
  - High rate of loss capitalization
- Withstand against system over-voltages
  - Reliable high voltage insulation design
- Short circuit withstand design
  - Analysis of dynamic behavior
  - Controlled manufacturing practices
- Elimination of hot spots in
  - Core, windings and structural parts
- Robust structural design
  - Seismic withstand, transport induced stresses
- Lower noise levels
Problems against which transformer should be guarded

- Overfluxing conditions
- Part winding resonance
- Very fast transient overvoltages related to GIS switching
- Static electrification phenomenon
- Ferroresonance
- Power system harmonics
- Geomagnetic disturbances
Magnetic Circuit

- Trends
  - lower thickness material grades
  - domain refined grades
  - step lap joint
  - boltless yokes

Building Factor

- Defined as

  
  \[
  \text{Building factor} = \frac{\text{Built transformer core loss (watts/kg)}}{\text{Material Epstein core loss (watts/kg)}}
  \]

- Building factor is generally found to increase with improvement in material grade
  - penalty for deviation from grain orientation is higher
  - expected loss reduction with better grades may not be obtained
  - experimental / test data should be used
• Building factor is also a function of:
  - core construction
  - type of core joint
    - Step-lap joint
    - Mitred joint
  - number of laminations per lay
  - overlap length
  - angle of overlap
  - gaps at joints
  - operating flux density
  - proportion of corner weight
Types of Core Construction

- Three-phase three-limb
- Three-phase five-limb
- Single-phase three-limb
- Single-phase two-limb
- Single-phase four-limb
- The choice of construction is influenced by
  - user’s specifications, manufacturing limitations, transport restrictions
  - required zero-sequence characteristics
- Bolt-less yoke construction
  - better core utilization
  - core loss reduction
  - yoke laminations can be epoxy bonded
Overexcitation conditions

- Choice of flux density depends on specified overexcitation conditions
- Commonly specified overexcitation conditions are:
  110% or 115% continuous, 125% for 1 minute,
  140% for 5 seconds, 150% for 1 second

3-D FEM model for analysis under overexcitation
Flux distribution at 110% overexcitation condition

Eddy currents in frame
Core temperature rise

- Design of optimum number of cooling ducts
- Requires determination of temperature profile under most
- For most accurate temperature rise estimation, 3-D FEM analysis required with consideration of anisotropic thermal properties
- The limits of temperature rise for core surface and interior portions are different.
Windings

• Design of windings is influenced by the following considerations
  - dielectric
  - short circuit
  - electromagnetic (eddy / stray losses)
  - thermal

• These design considerations have often conflicting requirements:
  - conductor radius : dielectric Vs short circuit
  - paper covering : dielectric Vs thermal
  - conductor thickness : short circuit Vs eddy loss
  - first duct : thermal Vs dielectric
  - radial spacer width : thermal Vs short circuit

• Judicious selection of design parameters is necessary
Choice of winding conductor

- Design consideration
  - eddy loss reduction
  - improvement in space factor
  - cost-benefit analysis
Eddy loss evaluation and optimization

- Eddy loss per unit volume for a thin conductor

\[ P_E = \frac{\omega^2 B^2 t^2}{24 \rho} \]

where \( \omega \) is frequency in rad/sec
\( B \) is peak value of tangential leakage flux density on the conductor surface
\( t \) is dimension of the conductor perpendicular to field
\( \rho \) is resistivity of the conductor material

- Eddy loss per unit area for a thick conductor

\[ P_e \approx \frac{H_0^2}{\sigma \delta} \]
\( H_0 \) = peak value of incident field on surface
\( \sigma \) = conductivity, \( \delta \) = skin depth
• Eddy loss: due to axial and radial leakage fields
• 2-D field can be calculated by:
  - Roth’s method, Rabin’s method, 2-D FEM
• 3-D FEM: for most accurate calculations
  - efforts involved can get justified for very large transformers
• It is necessary to optimize the sum of copper loss and eddy loss
• Balancing of ampere-turn per unit height for LV and HV windings is essential to minimize radial field
• Tap zone needs special attention
• Use of different conductor widths may be done
Winding temperature rise

- Judicious choice for following parameters is required:
  - conductor paper covering
  - winding first duct
  - radial spacer dimensions
  - number and position of passes in guided oil flow arrangement
Types of windings

Continuous disk winding

Interleaved winding

Continuous disk winding with SER and SR
Shielded-conductor winding

- Series capacitance of various windings can be calculated by a method based on stored energy [*]

Numerical methods for Impulse voltage distribution

- Classical Standing Wave and Traveling Wave theories can be applied to only uniform windings
- Transformer windings are highly non-uniform
- Numerical methods are now used, which can simulate non-uniformities and multi-section windings easily due to fast and powerful computers
- A winding or set of windings is represented in equivalent circuit form
- Each winding is divided into a number of sections
- Winding capacitances and inductances are lumped
- Partial differential equations get converted into ordinary differential equations which can be solved by numerical methods
Computation of Impulse Voltage Distribution using State Variable Method

- The network differential equations for the equivalent circuit can be formulated in nodal form as

\[ \hat{C} \ddot{\hat{y}}(t) + \hat{G} \dot{\hat{y}}(t) + \hat{\Gamma} \hat{y}(t) = 0 \]

where, \( \hat{C} \) = nodal capacitance matrix with the inclusion of input node
\( \hat{G} \) = nodal conductance matrix with the inclusion of input node.
\( \hat{\Gamma} \) = nodal matrix of inverse inductance matrix with the inclusion of input node.

\( \hat{y}(t) \) = Output vector of node voltages including input node.
• State space model:

General state space model of differential equation (without input node) is

\[
\dot{X} = AX + Bv
\]
\[
y = SX + Dv
\]

where, \( X \) = state vector of state space
\( A, S \) = matrices of constant coefficients
\( B, D \) = column matrices of constant coefficients
\( v \) = input vector of applied impulse voltage
\( y \) = output vector of node voltages

The solution of state space equations is

\[
X(t) = e^{At} X(0^-) + \int_{0^-}^{t} e^{A(t-\tau)} B x(\tau) d\tau
\]
Simulation result

- The voltages calculated for various nodes for a particular winding are:
Simulation result Continued . . .

- The results of State variable method, SEQUEL and SPICE at node 6
Part winding resonance

- Can be triggered by line fault
- Waveshape of overvoltage seen at transformer terminals is generally of non-standard shape; hence normal tests (LI, SI, PF) will not ensure checking of withstand against part winding resonance
- Most of the failures reported in literature involved failure of tap windings (being inner parts, higher frequencies cannot penetrate; tap winding is subjected to lower frequency of few tens of kHz)
- Interleaving of tap winding may not always be best option
- If tap winding natural frequency is closer to excitation frequency, part winding resonance can occur which results in very high local voltages
- The transformer winding natural frequencies are generally in the range of $5 \text{ kHz}$ to few hundred kHz
- The natural frequency of the line is given by $\frac{\nu}{4L}$ where $\nu$ is the wave propagation velocity (300 for overhead transmission lines and 100 for cables) and $L$ is length of line in meters (distance of a transformer from the point where a switching operation or ground fault occurs)
- Placement of breaker can be accordingly decided
Insulation design

- As voltage rating increases, insulation design becomes the most important aspect of power transformers

- Comprehensive design verification is essential for reliability and optimization

- Pressure on designers to reduce material content of which insulation is a major component

- Margins between withstand levels and working stress levels are reducing

- It is important to accurately estimate stress levels for various critical electrode configurations inside the transformer
• **Major insulation:**
  - Insulation between windings
  - Insulation between winding to core (limb/yoke)
  - Insulation between coil and tank
  - Insulation between high voltage leads and earth parts

• **Minor insulation:**
  - Insulation between turns / discs

**Four types of tests:**

- Lightning impulse test
- Switching impulse test
- Short duration power frequency test
- Long duration test with PD measurement
• Conversion of test levels to one equivalent test level called as Design Insulation Level
  - generally taken as short duration one-minute power frequency test

<table>
<thead>
<tr>
<th>Test type</th>
<th>Multiplication factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lightning impulse level</td>
<td>~ 0.44</td>
</tr>
<tr>
<td>Switching impulse level</td>
<td>~ 0.55</td>
</tr>
<tr>
<td>Long duration power frequency level</td>
<td>~ 1.25</td>
</tr>
</tbody>
</table>


The basis for using a certain value or range of these factors is explained in:

Design of oil ducts

- Sub-division of oil duct increases kV/mm withstand stress
- Barrier against propagation of discharge streamer in oil
- Electric stress is inversely proportional to permittivity
- Barriers should be as thin as mechanically possible, otherwise there will be more stress in oil
- First duct should be as small as possible with due considerations to adequate cooling requirements
• Average stress in HI-LO gap is generally designed to be in the range of 5.5-6 kV/mm

\[ E_{av} = \frac{DIL}{(HILO \ gap - 0.5 \times solid \ insulation)} \]

• Manufacturing tolerance is added based on variations in material dimensions on the shop-floor

• For winding to main limb or end limb, value of stress designed is lower or use of electrostatic shield is necessary to shield sharp corners
• It is important to know contribution and significance of various factors affecting stress levels

• Detailed FEM analysis is desirable while doing optimization
End insulation Design

- Sub-division of oil duct from HV winding to LV winding and yoke is done so that margin in each of the resulting oil ducts is approximately same.

- Use of contoured angle rings along equipotential lines to minimize creep stress.
Contoured angle rings

Non-contoured angle rings

• Select a contour as shown in the earlier figure

• Find cumulative stress values in each oil duct and compare with the reference withstand equation:

\[ E_{oil} = 75d_1^{-0.38} \text{ kV/cm} \]

Cumulative creep stress calculation

(i) Note down the voltage values along the pressboard at the different points (e.g., at every 1 or 2 mm steps)
(ii) Note down the highest stress point, i.e., the point along the pressboard at which the stress is highest
(iii) Determine on which side of the highest stress point the field is higher and extend the path by one spatial step in that direction
(iv) Find the cumulative stress when the path is extended in either direction and choose a path extension in the direction yielding higher cumulative stress
(v) Repeat the above step number (iv) until the complete creepage path along the angle ring is encompassed
(vi) Withstand for each of these creepage distances is calculated by,

\[ E_{cr} = 15 \times d^{-0.37} \text{ kV/mm} \]

**High voltage lead clearances**

- Oil and paper stresses can be calculated by analytical formulae or FEM analysis
- Stressed oil volume (between max. stress and 90% of max. stress) is then calculated
• The 50% power frequency breakdown probability stress can be determined by
\[ E_{50} = 11.5 (SOV)^{1/9.5} + 2.5 \] kV/mm

where SOV is in cm³


• The safe withstand value is lower than \( E_{50} \) value and is influenced by risk of failure and quality of oil processing during manufacture.

• Different manufacturers will generally have different safety margins based on experience

• Barriers are generally put between lead and earth, close to lead. Extra advantage due to barriers may not be considered as they may not be along equipotential lines
Stray loss evaluation and control

- Stray losses in windings
  - Eddy current losses
  - Circulating current losses

- Stray losses in structural components:
  - Flitch-plate
  - Frame
  - Tank
  - Lead terminations

- Stray losses due to leakage field

- Stray losses due to high-current field

- Individual stray losses cannot be separated from load loss

- Experimental limitations of measuring losses in structural components

- Transformer is a complex three-dimensional structure with materials having nonlinear properties
Factors affecting stray losses

Eddy loss per unit surface area of a plate, subjected to (on one of its surfaces) a magnetic field of $H_{\text{rms}}$

$$P_e = \frac{H_0^2}{2\sigma \delta} = \sqrt{\frac{\omega \mu}{2\sigma}} \ H_{\text{rms}}^2$$

Hence, the total power loss in a steel plate with a permeability can be given in terms of the peak value of the field ($H_0$) as

$$P = a_1 \iint_S \sqrt{\frac{\omega \mu_s}{2\sigma}} \ \frac{H_0^2}{2} \ ds$$

where $a_1$ is the linearization constant
Factors affecting stray losses

Effect of surface excitation:
- Tangential (e.g. bushing mounting plate)
- Radial (e.g. tank)

Effect of type of material
- magnetic (mild steel)
- high resistivity non-magnetic material (stainless steel)
- high conductivity non-magnetic material (aluminum)

The thickness of material should be judiciously selected depending upon the material properties

The thickness of stainless steel structural component should be as small as possible from the stray loss considerations.
Tank loss

- Its a 3-D problem, very difficult to exactly estimate
- 2-D FEM or semi-analytically numerical methods like RNM-3D are used
- Can be very high if not properly shielded

Guidelines for shielding with magnetic shunts:
1. They can be of CRNGO or CRGO laminations.
2. Width can be maximum of around 200 mm
3. Gap between 2 shunts placed on tank should be as minimum as mechanically possible.
4. Thickness of shunts depends on amount of flux collected.
5. Length of shunt should be ideally from top yoke centre to bottom yoke centre. Shunt should be vertically continuous (one piece without any gap)
6. Shunts should cover at least 67% of area in front of windings.

Two types of shunts
- Flat
- Edgewise: better but the effect can be noticeable only for large power transformers
Width-wise    Edge-wise

Tank shunts
Copper or aluminum shielding on tank

- Used for odd shapes of tank
- Shields should have sufficient thickness
- Shields must be continuous as far as possible
- Path of reflected flux should be studied

Frame loss

- For low distribution transformers, go and return arrangement for leads should be used
- Frame of stainless steel is generally not recommended

High current loss: How to reduce:
- Use of non-magnetic material or electromagnetic shielding
- Placement of high current bars thickness-wise with respect to plate
**Flitch plate loss**

- Loss may be low but loss density can be very high resulting into hot spots
- Types of flitch plate: Mild steel, Mild steel with slots, Stainless steel and Laminated

![Diagram of different types of flitch plates](http://webs.uvigo.es/arwtr04)
Mild Steel Flitch Plate

 challenger and strategies in transformer design

by Prof. S.V. Kulkarni
Bushing mounting plate
<table>
<thead>
<tr>
<th>Current (Amperes)</th>
<th>Analytical (Watts)</th>
<th>3D FEM (Watts)</th>
<th>From Steady State Temperature Rise (Watts)</th>
<th>From Initial Temperature Rise (Watts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>56</td>
<td>66</td>
<td>65</td>
<td>58</td>
</tr>
<tr>
<td>2250</td>
<td>68</td>
<td>84</td>
<td>74</td>
<td>70</td>
</tr>
<tr>
<td>2500</td>
<td>81</td>
<td>103</td>
<td>95</td>
<td>93</td>
</tr>
<tr>
<td>2800</td>
<td>98</td>
<td>130</td>
<td>119</td>
<td>116</td>
</tr>
</tbody>
</table>
Short Circuit Forces and Withstand

- The basic equation for the calculation of electromagnetic forces is

\[ \overline{F} = L \overline{I} \times \overline{B} \]

where \( B \) is leakage field density vector, \( I \) is the current vector and \( L \) is the length of winding.

- **Radial force** in x-direction due to axial leakage flux density and **axial force** in y-direction due to radial leakage flux density, as shown in the figure.
- **Radial forces:**

  - The radial forces are acting outwards on the outer winding tending to stretch the conductor, producing a tensile stress (also called as hoop stress).
  
  - The radial forces are acting inwards on the inner winding tending to collapse or crush it, producing a compressive stress.
  
  - The average hoop stress can be considered as uniform over the entire disk winding.
  
  - In a layer / helical winding, the average hoop stress is not uniform, i.e., it is highest for the innermost layers and it decreases towards the outer layers.
Axial forces:

- The axial forces due to radial fringing leakage field at winding ends are directed towards the center of winding from both ends for uniform ampere-turn distribution in windings with equal heights (ideal condition).
- The compressive force is maximum at the center of the windings.
- Both the inner and outer windings experience compressive forces with no end thrust on clamping structures at winding ends.
- For asymmetrical axial winding disposition (winding electromagnetic centres not at same level), axial forces on two windings are in opposite direction and they tend to increase the asymmetry.
- Strict sizing and dimension control required during processing and assembly of windings.
Dynamic Behavior under Short-Circuits

- The transformer windings along with supporting clamping structure form a mechanical system having mass and elasticity.
- The applied electromagnetic forces are oscillatory in nature and act on the elastic system comprising of winding conductors, insulation system and the clamping structure.
- The forces are dynamically transmitted to various parts of the transformer and can be quite different from the applied forces.
- To find out the stresses and displacements produced by short circuit forces, it is necessary to analyze the system dynamically.
- Method for calculating dynamic response is quite complex
Different numerical methods are available for determining the dynamic response of transformer under short circuit conditions.

Due to lack of precise knowledge of dynamic characteristics of various materials used in transformers, the methods for assessing dynamic performance under short circuits have not been yet perfected.

The dynamic calculations can certainly increase theoretical knowledge of the whole phenomenon, but it may be difficult to use the results of analysis for designing the transformer.

It is fairly easy to calculate natural frequencies of windings and check the absence of resonance.

Hence, established static calculations along with natural frequency determination could form a basis of short circuit strength calculations until the dynamic analysis is perfected and standardized.
Failure Modes due to Radial Forces

- Radial collapse of inner winding is common, whereas outwards bursting of outer winding generally does not take place.
- If a winding is tightly wound, conductors in radial direction can be assumed to have uniform tensile stress.
- Most of the space in radial direction is occupied with copper (except for the small paper covering on conductors): high stiffness to mass ratio.
- The natural frequency is much higher than the excitation frequency and hence chances of resonance are remote.
- The chances of winding failing with tensile hoop stress are unlikely if conductor with a certain minimum 0.2% proof strength is used.
- The 0.2% proof stress is defined as that stress value which produces a permanent strain of 0.2%.
Failure Modes due to Radial Forces Continued . . . .

- Windings subjected to compressive stresses:
  - Inner winding conductors, which are subjected to the axial compressive load, may fail due to bending between supports or buckling.
  
  - Forced buckling (1): occurs when the winding cylinder has significant stiffness as compared to winding conductors.
  - Free buckling (2): this kind of failure occurs mostly with lower thickness of winding cylinders.

- An adequate number of winding supports need to be provided to give strength to the winding against the radial forces.
Failure Modes due to Axial Forces

- If a layer winding is not wound tightly, some conductors may just axially pass over the adjacent conductors, which may result into damage to conductor insulation and a eventual turn-to-turn fault.

- If winding is set into vibration under the action of axial forces, conductor insulation may get damaged due to relative movement between winding and axial insulation spacers.

- There could be deformation of end clamping structure and windings due to high axial end thrust.

- To maintain effective pressure on windings a clamping ring made of stiff insulating material (pre-compressed board or permawood) is used.

- There are two principal types of failures, viz. bending between blocks and tilting.
Effect of Pre-Stress

- Pre-stress is the clamping pressure applied on windings, after completion of core-coil assembly.
- It has significant impact on the response of windings during short circuit.
- It increases the stiffness of windings thereby increasing the mechanical natural frequencies of windings.
- It reduces the oscillatory forces acting on the insulation.
- With increase in value of pre-stress, the displacements also decrease.
- The value of pre-stress should be judiciously chosen depending upon the characteristics of core-winding assembly.
- The chosen value of pre-stress must get maintained during the entire life of a transformer.
Short-Circuit Withstand

- General guidelines and precautions that can be taken at specification, design and production stages of transformers for improvement in short-circuit withstand are:
  - Windings should be made of high grade proof-stress copper instead of annealed copper.
  - Use of epoxy bonded CTC in place of strip conductors on large transformers.
  - Use of lower current densities for critical transformers.
  - Matching of AT/mm of LV and HV windings along their height.
  - Supporting inner winding with thicker insulating cylinders.
  - Ensuring tight-fitting wooden dowels on core in close contact with the insulating cylinders.
  - Estimation of natural frequencies of windings and ensuring that there is no excited resonance.
Short Circuit Withstand Continued . . . .

- Strictly controlled manufacturing process for the windings.
- Elastic stabilization of windings and tight tolerances concerning winding lengths under specified clamping force and relative positioning of windings.
- Core and coil clamping structure consisting of robust and stiff parts duly fastened.
- Securing all winding exit leads and connections to bushings and tap-changers.
- Use of fibre glass cylinder for the inner winding.
- Vapour phase drying of coil assemblies in specific cases for better dimensional control.

Coupled Systems

Diagram:
- **POWER ELECTRONIC CIRCUIT**
  - Control signal
  - Voltage, power, current
- **CONTROL SYSTEM**
  - Sensors
  - Current, Magnetic flux
- **MAGNETIC FIELD**
  - Forces, Motional induction
- **ELECTRICAL FIELD**
  - Material Characteristics, Joule and iron losses
  - Material Characteristics, Dielectric losses
- **MOTION FIELD**
- **THERMAL FIELD**
  - Expansion, Compression
  - Fluid Friction
  - Cooling
- **FLUID FLOW FIELD**
About Coupled Fields

- Coupled field treatment for **accurate** and **realistic** analysis.
- Coupled field simulation plays important role to get an **optimum performance**.
- **Classification**: 1) weakly coupled 2) strongly coupled.
- **Weak coupling**: solution of one field as load to another field.
  - It is **flexible**, **modular** and **easy to use**.
- **Strong coupling**: Coupled field equations are solved simultaneously.
  - It **simultaneously** manages all **physical aspects** of considered fields.
  - It is used when interactions are **highly nonlinear** and the coupled fields have **comparable time constants**.
Thermo-Magnetic Simulation

• **Coupled Field Equations:**

\[
\nabla \cdot (\nu \nabla (A)) = -\sigma(T) \frac{V}{l} + \sigma(T) \frac{\partial A}{\partial t}
\]

\[
\nabla \cdot (k \nabla (T)) = -q(A, T) + m \frac{\partial T}{\partial t}
\]

where, \(k\) is the thermal conductivity, 

\(m\) is the mass density, \(c\) is the specific heat, \(q\) is the loss term.

• **Coupling Relations (Weakly Coupled):**

• Material temperature dependence:

\[
\sigma(T) = \frac{\sigma_{ref}}{1 + \alpha_{\sigma}(T - T_{ref})}
\]

• Loss calculation

\[
q(A, T) = \frac{1}{\Omega} \int_{\Omega_c} \sigma \left( \frac{V}{l} - \frac{\partial A}{\partial t} \right)^2 d\Omega
\]
Thermo-Magnetic Simulation

• Transformer losses consist of eddy loss, DC $I^2R$ loss, and additional stray losses in structural components.
• The calculated joule losses can be used as loads to the thermal model to predict the temperature rise.
• The winding eddy loss and stray losses increase due to current harmonics.

\[
A(t) = \sum_{h=1}^{N} A_h(t)e^{j\omega t}
\]
Field-Circuit Coupling

- **Electromagnetic Model:**

\[ \nabla \times \left( \frac{1}{\mu} \nabla \times \mathbf{A} \right) = \mathbf{J}_0 - \sigma \frac{\partial \mathbf{A}}{\partial t} - \sigma \nabla V \]

- **Circuit Coupling:**

Here, \( L_{\text{ext}} \) and \( R_{\text{ext}} \) are external inductance and resistance respectively.
Field-Circuit Coupling: Conductor models

- **Stranded Conductor:**
  \[
  J_{str} = \frac{N_{str} I_{str}}{A_{str}}
  \]

- **Solid Conductor:**
  \[
  J_{sol} = \sigma \frac{V_{sol}}{l_{sol}} - \sigma \frac{\partial A}{\partial t}
  \]

- **Global System of Equation:**
  \[
  \begin{bmatrix}
  Q & 0 & 0 \\
  B^T & 0 & 0 \\
  0 & 0 & -M
  \end{bmatrix}
  \begin{bmatrix}
  \dot{A} \\
  \dot{g} \\
  \dot{i}
  \end{bmatrix}
  +
  \begin{bmatrix}
  C & B & 0 \\
  0 & S & P \\
  0 & P^T & -R
  \end{bmatrix}
  \begin{bmatrix}
  A \\
  g \\
  i
  \end{bmatrix}
  =
  \begin{bmatrix}
  f \\
  0 \\
  U
  \end{bmatrix}
  \]

  Here, \(Q, B, M, C, S, P\) and \(R\) are the matrices of the constant coefficients, \(f\) and \(U\) are the source terms, and \(g\) is the voltage gradient \((V/l)\).
Transient Simulation of Transformer

\[ \nabla \times \frac{1}{\mu} \nabla \times \mathbf{A} - \nabla \frac{1}{\mu} \nabla \cdot \mathbf{A} + \sigma \nabla V + \sigma \frac{\partial \mathbf{A}}{\partial t} = 0 \]

\( \nabla \cdot \left( -\sigma \nabla V - \sigma \frac{\partial \mathbf{A}}{\partial t} \right) = 0 \quad \text{In eddy current region} \)

\[ \nabla \times \frac{1}{\mu} \nabla \times \mathbf{A} - \nabla \frac{1}{\mu} \nabla \cdot \mathbf{A} = \mathbf{J} \quad \text{In non-eddy current region} \]

\[ J = \frac{N_c}{S_c} i(t) \]

where \( N_c \) is the number of turns and \( S_c \) is the total cross-sectional area of the windings.
Half-Turn Effect in Transformers

http://webs.uvigo.es/arwtr04

http://webs.uvigo.es/arwtr04
Half-Turn Effect in Transformers

Leakage flux lines without half-turn (a) ANSYS (b) MATLAB code
Half-Turn Effect in Transformers

Flux lines without half-turn (a) ANSYS (b) MATLAB code

http://webs.uvigo.es/arwtr04
Half-Turn Effect in Transformers

Without half-turn (average flux density in core is 0.02T)

With half-turn (average flux density in core is 1.2T)
Split-Winding Transformer

Flux lines during the short-circuit (a) ANSYS (b) MATLAB code
Thank You
ARWtr 2004 Lecture: **CHALLENGES AND STRATEGIES IN TRANSFORMER DESIGN** by Prof. S.V.Kulkarni

[http://webs.uvigo.es/arwtr04](http://webs.uvigo.es/arwtr04)